DFS* and the Traveling Tournament Problem

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Traveling Tournament Problem

- Sports scheduling combinatorial optimization problem.
- Objective is to create double round robin tournament with minimal travel distance.
- To date, only smallest few instances have been solved to optimality.
- Most best solutions found with meta-heuristics.

Traveling Tournament Problem

- Input: *n* teams and associated distances.
- Output: Double round robin tournament.
- Two constraints: *at_most* and *no_repeat*.

Round / Team	1	2	3	4	5	6
A	@B	@C	@D	B	C	D
B	A	D	C	@A	@D	@C
C	@D	A	@B	D	@A	B
D	C	@B	A	@C	B	@A

Traveling Tournament Problem

- Objective: Minimize travel distance.
- Distances calculated individually for each team, then summed together.

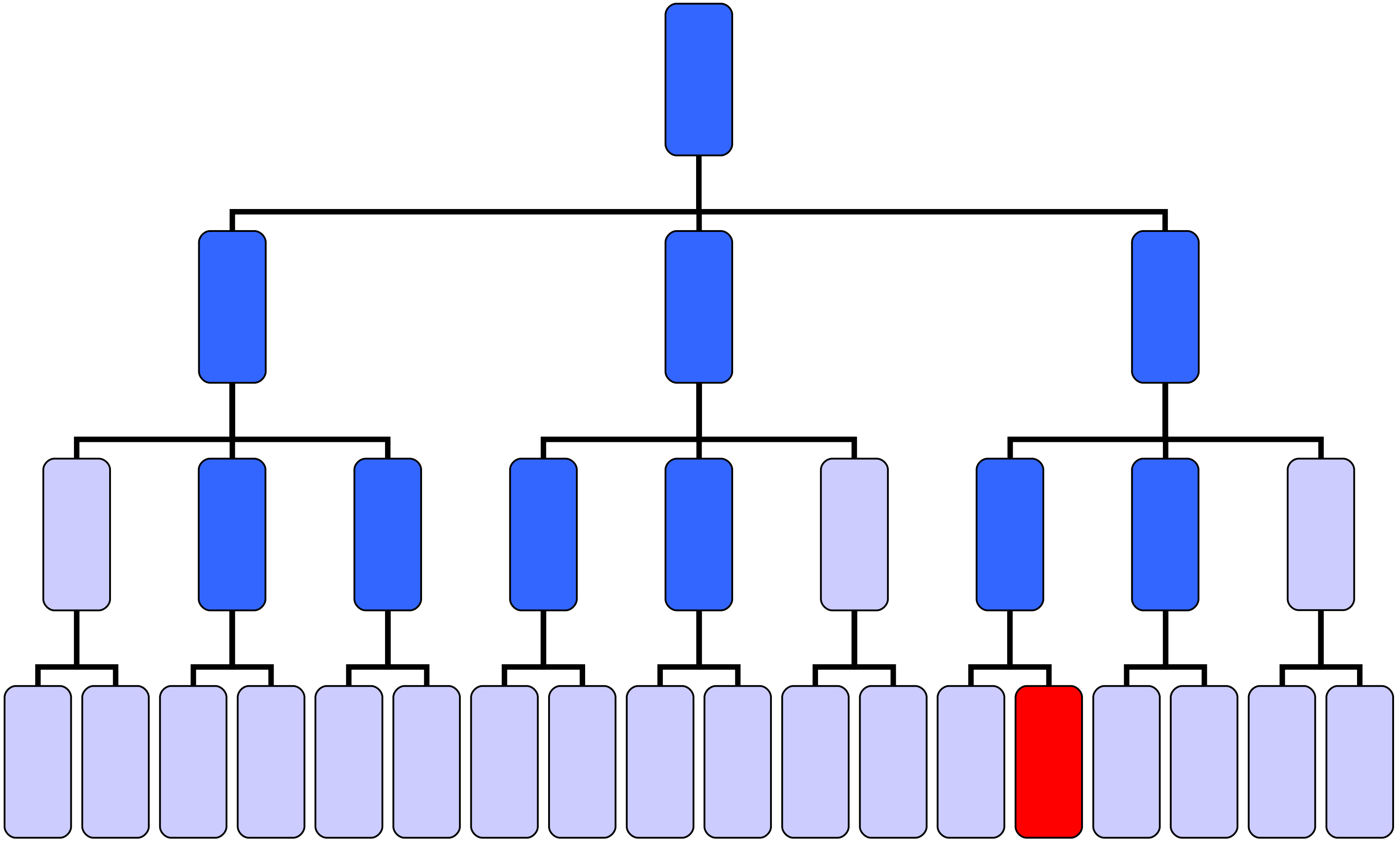
Team \ Round	1	2	3	4	5	6
A	@B	@C	@D	B	C	D
B	A	D	C	@A	@D	@C
C	@D	A	@B	D	@A	B
D	C	@B	A	@C	B	@A

DFS*

- Hybridization of IDA* and depth-first branch-and-bound.
- Also known as IDA*_CR and MIDA*.
- Each iteration, increase upper bound by greater amount than IDA*.
- Final iteration, after a solution is found, continue on as depth-first branch-and-bound.

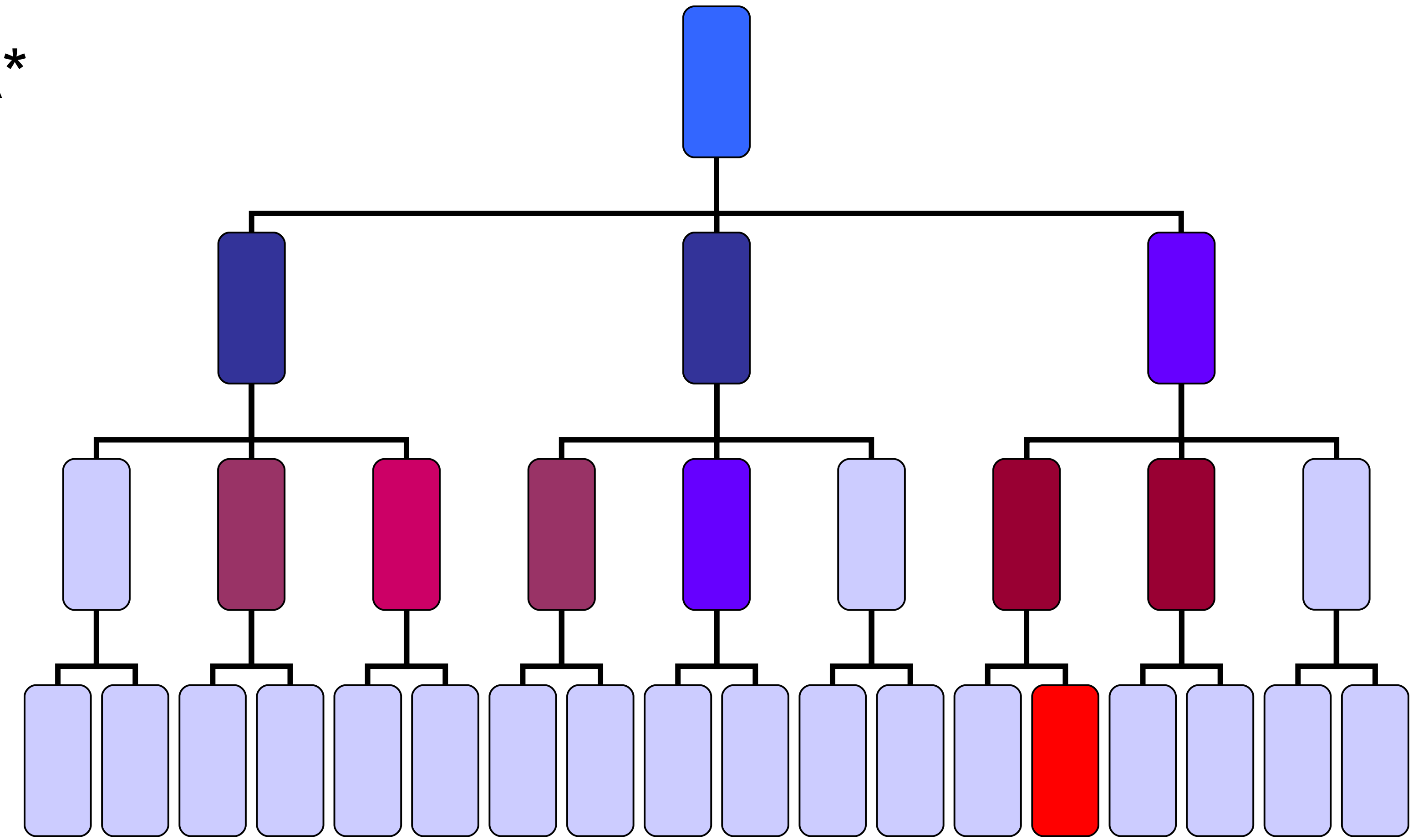
DFS*

- A*



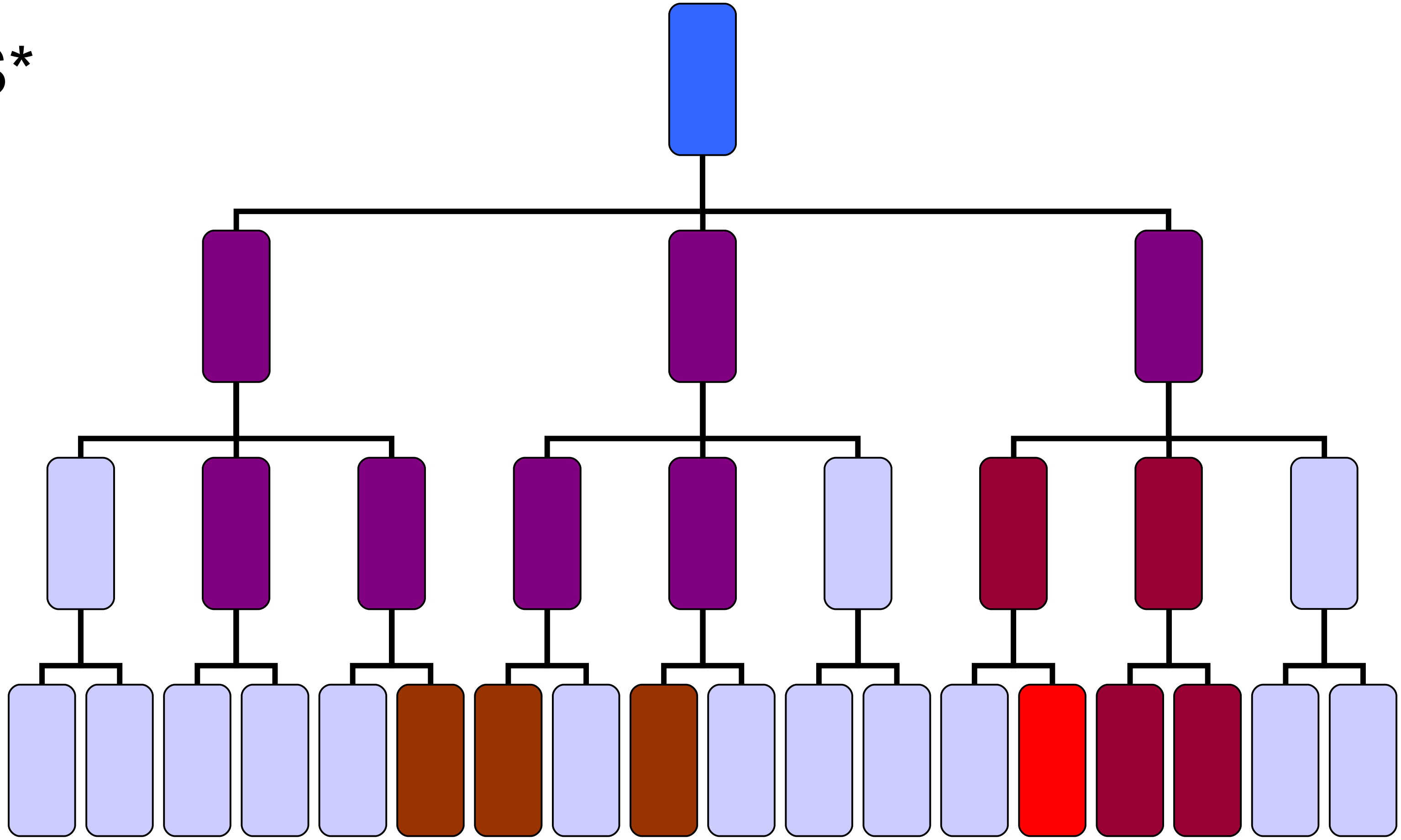
DFS*

- IDA*



DFS*

- DFS*



DFS* - Components

- Depth-First Search
- Memory & Heuristic Estimates
- Subtrees
- New Upper Bounds
- Symmetry
- Parallelization

Depth-First Search

- Pair up teams one round at a time from round 1 to *n*.
- Finish pairings teams within a round before moving to next round.

Round / Team	1	2	3	4	5	6
A	@B	@C				
B	A					
C	@D	A				
D	C					

Depth-First Search

- Easy to propagate constraints.
- Easy to calculate distance travelled so far.

Round / Team	1	2	3	4	5	6
A	@B	@C				
B	A					
C	@D	A				
D	C					

DFS* - Components

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Memory & Heuristic Estimates

- Heuristic estimates are minimal travel distance for each individual team.
- Sum individual estimates with distance traveled to get estimated total distance.

Team \ Round	1	2	3	4	5	6
A	@B	@C				
B	A					
C	@D	A				
D	C					

Memory & Heuristic Estimates

- Heuristic estimates expensive to calculate.
- DFS* uses minimal memory.
- Use available memory to store all heuristic estimates in a multi-dimensional matrix.
- Use two-level approach:
  - Upper level stores all estimates (index: O(n)).
  - Lower level stores current estimates (index: O(1)).

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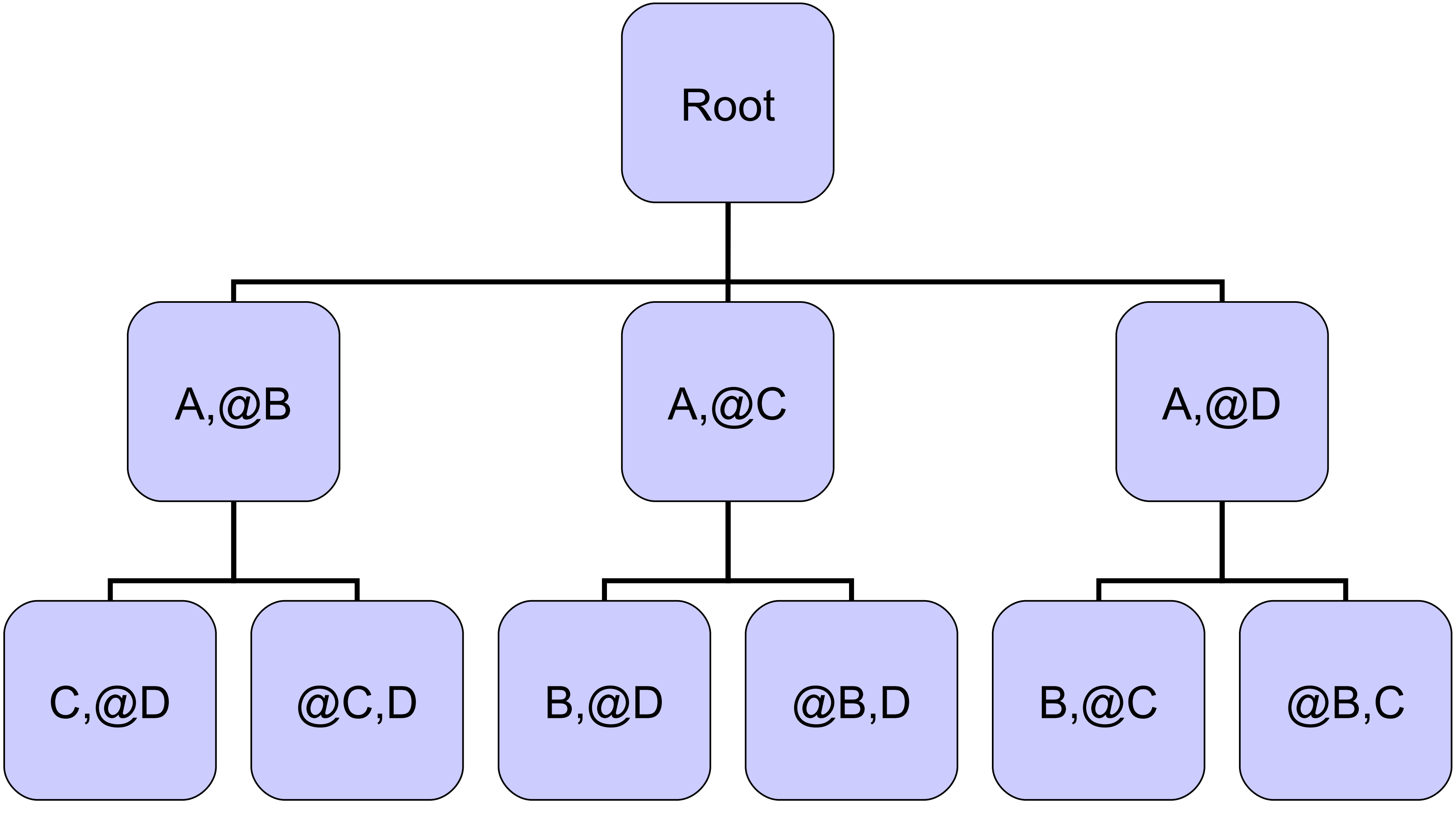
Subtrees



Subtrees

- Each subtree consists of the first 4 pairings of depth-first search.
- Order subtrees after each iteration so most promising are tried first in final iteration.
- Used for calculating new upper bounds.
- Allows for DFS* to be parallelized.

DFS* - Components

- Depth-First Search
- Memory & Heuristic Estimates
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- New Upper Bounds
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New Upper Bounds

- Based off of information from subtrees.
- Takes minimal lower bound of subtree which achieved deepest depth.
- Adds small extra cost associated with deepest depth and average distance in distance matrix.
- Purpose is to decrease number of iterations.

DFS* - Components

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- Parallelization

Symmetry

- Problem is horizontally symmetrical.
- Eliminate symmetry by using first team as pivot, check make sure # remaining away is greater than remaining number home games.

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A	@B	@C	@D	B	C	D
B	A	D	C	@A	@D	@C
C	@D	A	@B	D	@A	B
D	C	@B	A	@C	B	@A

DFS* - Components

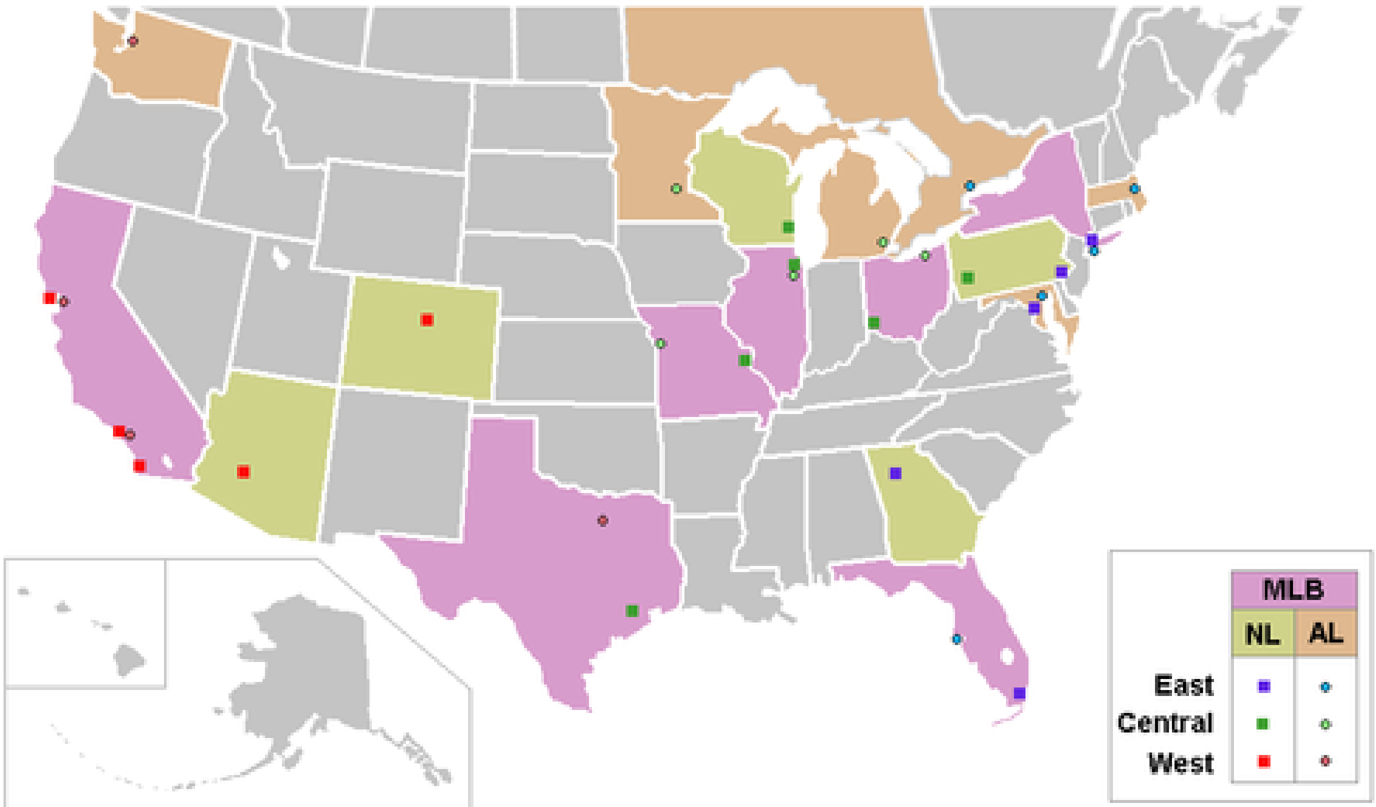
- Depth-First Search
- Memory & Heuristic Estimates
- Subtrees
- New Upper Bounds
- Symmetry
- Parallelization

Parallelization

- DFS* with subtrees ideal for parallelization: very few race conditions.
- Implemented on a shared-memory multi-cpu compute server.
- Each cpu will work on a single subtree at one time.

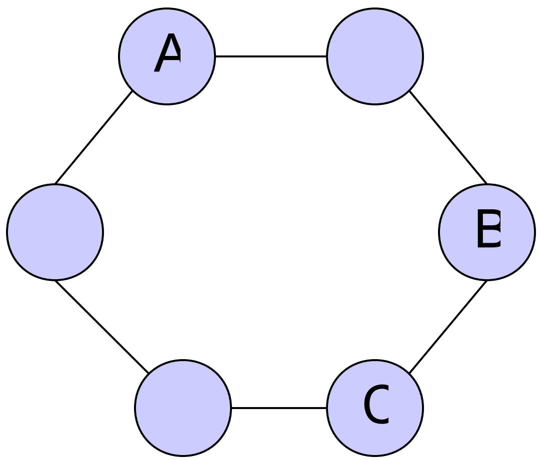
Problem Instances

- NL instances: Based on real world distances of NL teams in MLB.



Problem Instances

- CIRC instances: All teams placed on a circle, distance is minimal distance to another team going through neighbors.



Problem Instances

- Super14 instances: Based on real world distances of Super 14 Rugby League.

Performance

- Using memory reduced time on NL8 from ~94,000 seconds to ~400 seconds.
- Eliminating symmetry helped to improve performance for NL instances by almost half, had smaller impact with CIRC instances.
- Parallelization helped to further reduce running time, but not 100% efficient.

Comparison

	Irnich and Schrempp	Us
NL4	<0.3 secs	0.0 secs
NL6	<19 mins	0.98 secs
NL8	<18 hrs	262.42 secs
CIRC4	<0.2 secs	0.0 secs
CIRC6	<18 hrs	2.05 secs

Other results

- First to solve CIRC8, 337 seconds required across 4 processors.
- New lower bounds found for NL10, NL12, and CIRC10.
- Introduced Super 14 instances, solved team sizes 4 – 10, lower bounds only for 12 and 14 teams.

Future

- Look into distributed computing.
- Stronger heuristic estimates and better usage of memory.
- Further reduce symmetry with CIRC instances.
- Use pattern matching to improve constraint propagation.

Conclusions

- DFS* approach fastest to find known optimal solutions.
- Biggest impacts were storing heuristic estimates in memory and eliminating symmetry.
- DFS* can be easily parallelized, potential for distributed computing.